

# Modulation of the waterfall by a gauge field\*

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**ABSTRACT:** We present the first complete calculation of the curvature perturbation generated during the hybrid inflation waterfall, caused by the coupling of the waterfall field to a gauge field  $\mathbf{A}$  whose kinetic function  $f^2$  depends on the inflaton field. We impose an upper bound on the field  $\mathbf{W} \equiv f\mathbf{A}$  which ensures that it has a negligible effect before the waterfall. We confirm the claim of Soda and Yokoyama, that the perturbation  $\delta\mathbf{W}$  generates a statistically anisotropic spectrum and bispectrum, which could easily be observable. We also discover a new phenomenon, whereby the time-dependent ‘varyon’ field  $\mathbf{W}$  causes the inflaton contribution to vary during the waterfall. The varyon mechanism might be implemented also with a scalar field and might not involve the waterfall.

**KEYWORDS:** Primordial curvature perturbation.

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## 1. Introduction

During the waterfall of hybrid inflation, the perturbation of the waterfall field generates a contribution to the curvature perturbation. But its spectrum is proportional to  $k^3$  which almost certainly makes it negligible on cosmological scales [1]. For the waterfall to generate a contribution with a nearly flat spectrum, its onset should be modulated by (i.e. depend upon the value of) some field that is different from both the inflaton and the waterfall field, and whose perturbation has a nearly flat spectrum.

Such modulation was first considered in [2] using a scalar field (and further explored in [3, 4, 5, 6]). Then the Soda and Yokoyama [7] used instead a  $U(1)$  gauge

field.<sup>1</sup> In this paper we give the first complete treatment of that case. The gauge field  $\mathbf{A}$  has a kinetic function  $f^2$  that depends on the inflaton field. We impose a condition ensuring that  $\mathbf{W} \equiv f\mathbf{A}$  has a negligible effect before the waterfall, and take into account both the perturbation of  $f$  and the possible time-dependence of  $\mathbf{W}$ . We confirm the claim of [7] that the *perturbation*  $\delta\mathbf{W}$  can generate a statistical anisotropic contribution to  $\zeta$ , at a level which could easily be observed. We also find a new effect, which is that the *time-dependence* of  $\mathbf{W}$  can cause a significant variation in the (statistically isotropic) inflaton contribution to  $\zeta$ . We dub this new effect ‘the varyon mechanism’ and note that the varyon field might not be a gauge field and might not act during the waterfall.

We will take for granted the main ideas of modern cosmology described for instance in [9], and use the notation and definitions of [10, 9]. The unperturbed universe has the line element

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j. \quad (1.1)$$

In the perturbed universe, we can choose a slicing (fixed  $t$ ) and threading (fixed  $\mathbf{x}$ ), and write for a given quantity  $g(\mathbf{x}, t) = g(t) + \delta g(\mathbf{x}, t)$ . A different slicing with a time displacement  $\delta t(\mathbf{x}, t)$  gives a different perturbation  $\tilde{\delta}g$ . If  $g$  is rotationally invariant we have to first order

$$\tilde{\delta}g(\mathbf{x}, t) - \delta g(\mathbf{x}, t) = -\dot{g}(t)\delta t(\mathbf{x}, t). \quad (1.2)$$

We will invoke this ‘gauge transformation’ without comment. In most cases  $g$  is homogeneous on one of the slicings.

We denote the Fourier component by  $\delta g_{\mathbf{k}}(t)$  where  $\mathbf{k}$  is the coordinate wavenumber. Cosmological scales (probed directly by the CMB anisotropy and galaxy surveys) range from  $k = k_0 \equiv (aH)_0$  to  $k \sim e^{15}k_0$ , where  $H \equiv \dot{a}/a$  and  $(aH)_0$  is evaluated at the present epoch so that  $a_0/k_0$  is about the size of the observable universe. A scale is ‘outside the horizon’ if  $k < aH$ . Inflation corresponds to  $\epsilon_H < 1$  where  $\epsilon_H \equiv -\dot{H}/H^2$ . Cosmological scales leave the horizon during inflation and enter the horizon during the radiation-dominated era leading to Big Bang Nucleosynthesis (BBN).

## 2. The curvature perturbation $\zeta$

### 2.1 Definition and $\delta N$ formula

To define  $\zeta$  one smoothes the metric on a super-horizon scale, and adopts the co-moving threading and the slicing of uniform energy density  $\rho$ . Then [11, 12]

$$\zeta(\mathbf{x}, t) \equiv \delta[\ln a(\mathbf{x}, t)] = \delta[\ln(a(\mathbf{x}, t)/a(t))] \equiv \delta N(\mathbf{x}, t), \quad (2.1)$$

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<sup>1</sup>This is extended to the non-Abelian case in [8].

where  $a(\mathbf{x}, t)$  is the locally defined scale factor (such that a comoving volume element is proportional to  $a^3(\mathbf{x}, t)$ ). The number of  $e$ -folds of expansion  $N(\mathbf{x}, t, t_*)$  starts from a slice at time  $t_*$  on which  $a$  is unperturbed (‘flat slice’) and ends on a uniform  $\rho$  slice at time  $t$ . Since the expansion between two flat slices is uniform,  $\delta N$  is independent of  $t_*$ .

By virtue of the smoothing, the energy conservation equation is valid locally:

$$\dot{\rho}(t) = -3 \frac{\partial a(\mathbf{x}, t)}{\partial t} (\rho(t) + P(\mathbf{x}, t)). \quad (2.2)$$

In consequence,  $\dot{\zeta} = 0$  during an era when  $P(\rho)$  is a unique function. The success of the BBN calculation shows that  $P = \rho/3$  to high accuracy just before cosmological scales start to enter the horizon. Then  $\zeta$  has a time-independent value  $\zeta(\mathbf{x})$  that is strongly constrained by observation. Within observational errors it is gaussian and statistically isotropic. Its spectrum is nearly independent of  $k$ , with [13]

$$\mathcal{P}_\zeta(k) \simeq (5 \times 10^{-5})^2 \quad (2.3)$$

$$n(k) - 1 \equiv d \ln \mathcal{P}_\zeta / d \ln k \simeq -0.032 \pm 0.012. \quad (2.4)$$

(The result for  $n(k)$  assumes that it has negligible scale dependence. It also assumes a tensor fraction  $r \ll 10^{-1}$ , which will soon be tested by PLANCK [14].) For the reduced bispectrum [13]  $f_{\text{NL}}$ , current observation give  $|f_{\text{NL}}| \lesssim 100$  and barring a detection PLANCK will give  $|f_{\text{NL}}| \lesssim 10$ . For  $f_{\text{NL}}$  to ever be observable we need  $|f_{\text{NL}}| \gtrsim 1$ .

We will work to first order in  $\zeta$ , so that

$$\zeta(\mathbf{x}, t) = H(t) \delta t_{f\rho}(\mathbf{x}, t), \quad (2.5)$$

where  $\delta t_{f\rho}$  is the time displacement from the flat slice to the the uniform- $\rho$  slice. A second-order calculation of  $\zeta$  is needed only to treat very small non-gaussianity corresponding to  $|f_{\text{NL}}| \lesssim 1$ .

We adopt the usual assumption, whereby  $N(\mathbf{x}, t, t_*)$  is determined by the values of one or more fields  $\phi_i(\mathbf{x}, t)$ , evaluated during inflation at an epoch  $t_*$ .<sup>2</sup>

$$N(\mathbf{x}, t) = N(\phi_1^*(\mathbf{x}), \phi_2^*(\mathbf{x}), \dots, t). \quad (2.6)$$

Defining the perturbations  $\delta\phi_i^*$  on a flat slice, one writes [11, 15]

$$\zeta(\mathbf{x}, t) = \sum N_i(t) \delta\phi_i^*(\mathbf{x}) + \frac{1}{2} \sum_{ij} N_{ij}(t) \delta\phi_i^*(\mathbf{x}) \delta\phi_j^*(\mathbf{x}) + \dots, \quad (2.7)$$

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<sup>2</sup>To be more precise,  $N$  will depend also on some of the masses and couplings in the action, and it may depend too on the values of any fields with negligible dependence on  $\mathbf{x}$  and  $t$  that have not time to reach their vacuum expectation values. That does not affect any of the following.

where a subscript  $i$  denotes  $\partial/\partial\phi_i^*$  evaluated at the unperturbed point of field space. The  $\phi_i$  are usually taken to be scalar fields, but it has been proposed [7, 10] that some or all of them may be components of a vector field.

On each scale  $k$ , the field perturbations are generated from the vacuum fluctuation at horizon exit and are initially uncorrelated. Ignoring scales leaving the horizon after  $t_*$  Eq. (2.7) defines a classical quantity  $\zeta$ . In general it depends on  $t$ , settling down to the observed quantity  $\zeta(\mathbf{x})$  by some time  $t_f$ . Since  $\zeta(\mathbf{x})$  is nearly gaussian, one assumes that Eq. (2.7) is dominated by one or more linear terms involving nearly gaussian scalar fields. With  $t_*$  chosen as the epoch of horizon exit for a scale  $k$  this gives

$$\mathcal{P}_\zeta(k, t) \simeq \sum N_i^2(t_*(k), t) \mathcal{P}_{\delta\phi_i^*}(k, t_*(k)) + \dots, \quad (2.8)$$

where the terms exhibited correspond to scalar fields, and the dots indicate vector field contributions [10]. Each contribution is positive.

## 2.2 Slow-roll inflation

Slow-roll inflation invokes Einstein gravity, and one or more scalar fields with the canonical kinetic term. The fields have practically gaussian perturbations, with  $\mathcal{P}_{\delta\phi_*} = (H/2\pi)^2$  at horizon exit. During single-field slow-roll inflation, only the inflaton  $\phi$  has significant variation. Its unperturbed value  $\phi(t)$  satisfies

$$3H\dot{\phi} \simeq -V'(\phi), \quad (2.9)$$

where the potential  $V$  satisfies

$$\epsilon \equiv \frac{1}{2}M_{\text{P}}^2(V'/V)^2 \simeq \epsilon_H \ll 1 \quad (2.10)$$

$$|\eta| \ll 1, \quad \eta \equiv M_{\text{P}}^2 V''/V, \quad (2.11)$$

giving  $\rho = 3M_{\text{P}}^2 H^2 \simeq V$ .

The perturbation  $\delta\phi_*$  generates a contribution  $\zeta_\phi$ . Since  $\phi(\mathbf{x}, t)$  is the only time-dependent field, the effect on  $N$  of its perturbation  $\delta\phi_*(\mathbf{x})$  can be removed by the time shift  $\delta t(\mathbf{x})$  which makes  $\phi_*$  homogeneous, which means that  $\zeta_\phi$  is time-independent. At first order,

$$\zeta_\phi(\mathbf{x}) = -(H/\dot{\phi})\delta\phi_*(\mathbf{x}), \quad (2.12)$$

and

$$\mathcal{P}_{\zeta_\phi}(k) \simeq \frac{1}{2\epsilon M_{\text{P}}^2} \left( \frac{H}{2\pi} \right)^2 \quad (2.13)$$

$$n_\phi(k) - 1 \equiv d\mathcal{P}_{\zeta_\phi}/d\ln k = 2\eta - 6\epsilon \simeq 2\eta \quad (2.14)$$

where the right hand sides are evaluated at horizon exit. The second equality of Eq. (2.14) is appropriate for small-field models [16] and it applies to the standard

hybrid inflation which we are going to consider. The contribution of  $\zeta_\phi$  to  $|f_{\text{NL}}|$  is  $[17] \lesssim 10^{-2}$ .

For multi-field slow-roll inflation, where two or more fields have significant variation during inflation, Eqs. (2.12)–(2.14) refer to the contribution of the field pointing along the inflaton trajectory at horizon exit. Field perturbations orthogonal to the (single- or multi-field) trajectory give no contribution to  $\zeta$  at horizon exit, but may contribute later. (This may occur during slow-roll inflation in a multi-field model, or during the waterfall, or after inflation through a curvaton-type mechanism.) We therefore have

$$\mathcal{P}_{\zeta_\phi}(k) \lesssim \mathcal{P}_\zeta(k), \quad (2.15)$$

where  $\mathcal{P}_\zeta(k) \simeq (5 \times 10^{-5})^2$  is the observed quantity. The tensor fraction therefore satisfies

$$r \leq 16\epsilon, \quad (2.16)$$

with  $\epsilon$  evaluated when  $k_0$  leaves the horizon.<sup>3</sup> This leads [16] to what has been called the Lyth bound, on the variation  $\Delta\phi$  of the inflaton field after  $k_0$  leaves the horizon,  $r \lesssim 10^{-1} (\Delta\phi / M_{\text{P}})^2$ . For the tensor fraction to be detectable in the foreseeable future one needs  $r \gtrsim 10^{-3}$  [18], which is impossible in a small-field model ( $\Delta\phi \lesssim 10^{-1} M_{\text{P}}$ ).

### 3. The model

#### 3.1 Hybrid inflation

We are interested only in the era starting with horizon exit for  $k_0$  and ending with the onset of the waterfall. The relevant part of the action is taken to be

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{\text{P}}^2 R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} - V \right], \quad (3.1)$$

$$V(\phi, \chi, A) = V_0 + \Delta V(\phi) + \frac{1}{2} m^2(\phi, A) \chi^2 + \frac{1}{4} \lambda \chi^4 \quad (3.2)$$

$$m^2(\phi, A) \equiv h^2 A^2 + g^2 \phi^2 - m^2. \quad (3.3)$$

with  $F_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$  and  $B_\mu$  a  $U(1)$  gauge field. To fix the normalization of  $f$ , we set  $f = 1$  at a time  $t_{\text{w}}$  just before the waterfall begins.

Following [10] we use the gauge with  $B_0 = \partial_i B_i = 0$ , and work with  $A_i \equiv B_i/a$  which is the field defined with respect to the locally orthonormal basis (as opposed to  $B_i$  which is defined with respect to the coordinate basis). The raised component is  $A^i = A_i$  (as opposed to  $B^i = B_i/a^2$ ). We also define the canonically normalized field  $\mathbf{W} \equiv f \mathbf{A}$ . The waterfall field  $\chi$  is supposed to be the radial part of a complex field which is charged under the  $U(1)$ , generating the first term of Eq. (3.3).

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<sup>3</sup>This follows from the definition  $r \equiv \mathcal{P}_h(k)/\mathcal{P}_\zeta(k)$  with  $k \simeq k_0$ , and the prediction  $\mathcal{P}_h(k) = (8/M_{\text{P}}^2)(H/2\pi)^2$  with  $H$  evaluated at horizon exit.

We are going to impose Eq. (3.12), which ensures that  $\mathbf{W}$  has a negligible effect before the waterfall. Then, assuming suitable values for the parameters and field values, Eq. (3.1) gives what has been called [1] standard hybrid inflation [19, 20]. At each location, the waterfall begins when  $m^2(\phi, A)$  falls to zero. Before it begins, the waterfall field  $\chi$  vanishes up to a vacuum fluctuation which is set to zero, and we have slow-roll inflation with

$$V = V_0 + \Delta V(\phi) \simeq V_0. \quad (3.4)$$

We will take  $H$  to be constant which is typically a good approximation. In contrast with [7], we will not assume  $\Delta V(\phi) \propto \phi^2$ .

During the waterfall,  $\chi$  moves to its vev and then inflation ends. We will assume that the duration of the waterfall is so short that it can be taken to occur on a practically unique slice of spacetime. This requires [1]  $m \gg H$  and  $H \lesssim 10^9 \text{ GeV}$ .

### 3.2 Field equations with $f \propto a^\alpha$

To work out the field equations, most previous authors have taken  $f(\phi(\mathbf{x}, t))$  to be a function only of time with  $f \propto a^\alpha(t)$  (see however [21]). Taking spacetime to be unperturbed, the action (3.1) then gives for the unperturbed fields

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + V'(\phi(t)) = 0 \quad (3.5)$$

$$\ddot{\mathbf{W}}(t) + 3H\dot{\mathbf{W}}(t) + \mu^2\mathbf{W}(t) = 0, \quad (3.6)$$

where

$$\mu^2 \equiv H^2(2 + \alpha)(1 - \alpha). \quad (3.7)$$

By virtue of the flatness conditions (2.10) and (2.11), the first expression is expected to give the slow-roll approximation (2.9) more or less independently of the initial condition. Similarly, the second equation is expected to give the slow-roll approximation  $3H\dot{\mathbf{W}} \simeq -\mu^2\mathbf{W}$  if  $|\mu|^2 \ll H^2$ . This condition is assumed because the analysis would otherwise become much more complicated. It is equivalent to  $\alpha \simeq 1$  or  $-2$ .

The first order perturbations satisfy

$$\delta\ddot{\phi}_{\mathbf{k}}(t) + 3H\delta\dot{\phi}_{\mathbf{k}}(t) + [(k/a)^2 + V''(\phi(t))] \delta\phi_{\mathbf{k}}(t) = 0 \quad (3.8)$$

$$\delta\ddot{\mathbf{W}}_{\mathbf{k}}(t) + 3H\delta\dot{\mathbf{W}}_{\mathbf{k}}(t) + ((k/a)^2 + \mu^2)\delta\mathbf{W}_{\mathbf{k}}(t) = 0. \quad (3.9)$$

Keeping only super-horizon scales, Eqs. (3.6) and (3.9) and give

$$3H\dot{\mathbf{W}}(\mathbf{x}, t) \simeq -\mu^2\delta\mathbf{W}(\mathbf{x}, t). \quad (3.10)$$

The effect of the metric perturbation (back-reaction) on these equations vanishes in the limit where  $\phi$  and  $\mathbf{W}$  are constant [9, 22]. The assumption of unperturbed spacetime is therefore expected to be a good approximation.

In terms of  $W$ , the coupling  $h^2 A^2 \chi^2$  becomes  $\tilde{h}^2 W^2 \chi^2$ , where  $\tilde{h} \equiv h/f$ . We are setting  $f = 1$  when the waterfall begins at  $t = t_w$ . To generate  $\delta \mathbf{W}$  from the vacuum fluctuation, one assumes that  $\mathbf{W}$  is a practically free field while cosmological scales leave the horizon, corresponding to  $\tilde{h} \ll 1$ , or  $h \ll e^{-N(k)\alpha}$  where  $N(k)$  is the number of  $e$ -folds of inflation after horizon exit. With  $\alpha \simeq 1$  this would make  $h$  too small to have a significant effect. One therefore assumes  $\alpha \simeq -2$ .

The simplest supersymmetric hybrid inflation model [23] has  $\Delta V$  increasing logarithmically. Then  $f(\phi)$  increases exponentially. The same behaviour holds for the non-hybrid model with the full potential  $V \simeq 3M_{\text{P}}^2 H^2 \propto \phi^2$ . It might be reasonable in string theory [24], and which could correspond to an attractor [25].

### 3.3 Field equations with $f(\phi)$

In this paper we recognise that  $f$  is supposed to be a function of the inflaton field  $\phi$ , while retaining the assumption  $f \propto a^\alpha$  for the unperturbed quantity. Using the slow-roll approximation with  $\alpha = -2$  we have

$$\delta f = \frac{df}{d\phi} \delta\phi = \frac{df}{da} \frac{da}{dt} \frac{dt}{d\phi} \delta\phi = \frac{2}{\sqrt{2\epsilon} M_{\text{P}}} \delta\phi. \quad (3.11)$$

Since  $f$  is a function of  $\phi$ , the term  $-\frac{1}{4} f^2 F_{\mu\nu} F^{\mu\nu}$  in the action couples  $\phi$  and  $\mathbf{W}$  so that the right hand sides of Eqs. (3.5), (3.6), (3.8) and (3.9) are non-zero. We calculate them in the Appendix, and show that they are negligible if

$$\frac{\rho_W}{\epsilon\rho} = \frac{1}{2} \frac{\dot{W}^2}{\epsilon\rho} \simeq \frac{1}{2} \frac{\mu^2 W^2}{\epsilon V} \simeq \frac{1}{6} \frac{W^2}{\epsilon M_{\text{P}}^2} \ll 1, \quad (3.12)$$

where  $\rho_W$  is the energy density of  $\mathbf{W}$ . We will assume this condition. It implies  $\rho_W \ll \rho$ , which also ensures that  $\mathbf{W}$  has a negligible effect during slow-roll inflation. From Eqs. (2.3) and (2.13) the condition corresponds to

$$\frac{W(t)}{H} \lesssim 10^5 \left( \frac{\mathcal{P}_\zeta(k)}{\mathcal{P}_{\zeta_\phi}(k)} \right)^{1/2}, \quad (3.13)$$

where  $k$  is the scale leaving the horizon at time  $t$ .

### 3.4 The perturbation $\delta \mathbf{W}$

The evolution equation for  $\mathbf{W}(\mathbf{x}, t)$  is the same as that of a free scalar field with mass-squared  $\mu^2$ , and we are assuming  $|\mu|^2 \ll H^2$ . Treating the Fourier component  $\delta W_{\mathbf{k}}(t)$  as an operator and assuming the vacuum state well before horizon exit, one finds well after horizon exit the approximately scale-independent vacuum expectation value

$$\frac{k^3}{2\pi^2} \langle \delta W_{\mathbf{k}}^i(t) \delta W_{\mathbf{k}'}^j(t) \rangle = \left( \delta^{ij} - \hat{k}^i \hat{k}^j \right) \delta^3(\mathbf{k} + \mathbf{k}') \left( \frac{H}{2\pi} \right)^2 \left( \frac{k}{a(t)H} \right)^{\frac{2\mu^2}{3H^2}}, \quad (3.14)$$



where hats denote unit vectors. From Eq. (3.10), the operator  $\delta\mathbf{W}_{\mathbf{k}}$  has almost constant phase which means that  $\delta\mathbf{W}_{\mathbf{k}}$  can be treated as a classical quantity with this correlator.

The decomposition

$$\mathbf{W}(\mathbf{x}, t) = \mathbf{W}(t) + \delta\mathbf{W}(\mathbf{x}, t) \quad (3.15)$$

is made in some box of coordinate size  $L$  around the observable universe, with  $\mathbf{W}(t)$  the average within the box. After smoothing on a cosmological scale  $k$ , the spatial average of  $(\delta W)^2$  (evaluated within a region not many orders of magnitude bigger than the observable universe) is of order  $\ln(kL)(H/2\pi)^2$ . We assume  $W(t) \gg H$ , which is reasonable because  $W^2(t)$  at a typical position is expected to be at least of order the mean square of  $(\delta W)^2$  evaluated within a box with size  $M \gg L$  [26].

Including both the inflaton and  $\mathbf{W}$  and assuming that cubic and higher terms are negligible, Eq. (2.7) becomes [10]

$$\zeta(\mathbf{x}, t) = \zeta_\phi(\mathbf{x}, t) + \zeta_W(\mathbf{x}, t) + \sum_i \frac{1}{2} N_{\phi\phi}(t) [\delta\phi_*(\mathbf{x})]^2 + N_{\phi i} \delta\phi_*(\mathbf{x}) \delta W_i^*(\mathbf{x}) \quad (3.16)$$

$$\zeta_\phi(\mathbf{x}, t) \equiv N_\phi(t) \delta\phi_*(\mathbf{x}), \quad (3.17)$$

$$\zeta_W(\mathbf{x}, t) \equiv \sum_i N_i(t) \delta W_i^*(\mathbf{x}) + \frac{1}{2} \sum_{ij} N_{ij}(t) \delta W_i^*(\mathbf{x}) \delta W_j^*(\mathbf{x}), \quad (3.18)$$

where the subscripts on  $N$  denote partial derivatives evaluated on the unperturbed trajectory. We are assuming Eq. (3.12), which ensures that before the waterfall  $\zeta_W$  is negligible while  $\zeta(\mathbf{x}, t)$  is close to the time-independent quantity given by Eq. (2.12).

## 4. Effect of the waterfall on $\zeta$

### 4.1 End-of-inflation formula

Let us denote the contribution generated during the waterfall by  $\zeta_w$ .<sup>4</sup> To evaluate it, we assume that the waterfall happens very quickly so that it can be regarded as taking place on a single spacetime slice. Then [1, 2]

$$\zeta_w(\mathbf{x}) = H \delta t_{\rho\rho}(\mathbf{x}) = H \left[ \frac{\delta\rho_w(\mathbf{x})}{\dot{\rho}(t_w)} - \frac{\delta\rho_w(\mathbf{x})}{\dot{\rho}(t_+)} \right] \simeq H \frac{\delta\rho_w(\mathbf{x})}{\dot{\rho}(t_w)} \simeq H \delta t_{\rho w}. \quad (4.1)$$

In this equation,  $t_{\rho\rho}(\mathbf{x})$  is the proper time elapsing between a uniform- $\rho$  slice at time  $t_w$  just before the waterfall and a uniform- $\rho$  slice at time  $t_+$  just after the waterfall, while  $t_{\rho w}$  is the same thing with the final slice the waterfall slice itself.

This end-of-inflation formula actually holds if the waterfall slice is replaced by any sufficiently brief transition from inflation to non-inflation. In [1] it is invoked

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<sup>4</sup>Notice that we are using the subscript w to indicate the waterfall era. The quantity  $\zeta_w$  is the total change in  $\zeta$  during the waterfall, which as we shall see can be different from the contribution  $\zeta_W$  of  $\delta\mathbf{W}$ .

for the transition beginning *during* the waterfall, at the epoch when the evolution of  $\chi$  becomes non-linear. We are here applying it to the entire waterfall. It was first given [2] with  $A$  in Eq. (3.3) replaced by a scalar field. In [2] the slope of the potential in the  $A$  direction was assumed to be negligible corresponding to single-field hybrid inflation, and the same assumption was made in several later papers [3]. The assumption was relaxed in [4, 5, 6], corresponding to what has been called [5] multi-brid inflation. Following [7] we are here taking  $A$  to be the magnitude of a  $U(1)$  gauge field. One can also replace  $A$  by a non-Abelian gauge field [8, 27, 28].

## 4.2 Waterfall contribution: general formula

Instead of calculating  $\zeta_w$  directly, we calculate

$$\zeta(\mathbf{x}, t_+) = \zeta_w(\mathbf{x}) + \zeta_\phi(\mathbf{x}), \quad (4.2)$$

where  $\zeta_\phi(\mathbf{x})$  is given by Eq. (2.12). We do this first without specifying the function  $m^2(\phi, A)$  or the nature of  $A$ . We define  $\phi_w(A)$  by  $m^2(\phi_w, A) = 0$ . (If this equation has more than one solution  $\phi_w(A)$ , we choose one of them.) The waterfall occurs when  $\phi(\mathbf{x}, t) = \phi_w(\mathbf{x}, t)$ .

If  $t_{f\rho}(\mathbf{x})$  is the displacement from the flat slice at  $t_w$  to the uniform  $\rho$  slice at  $t_+$  we have  $\zeta(\mathbf{x}, t_+) = H\delta t_{f\rho}(\mathbf{x})$ . Making the good approximation  $\delta t_{f\rho} = \delta t_{fw}$ , where  $t_{fw}$  is the displacement from the flat slice to the waterfall slice, we have

$$\phi(\mathbf{x}, t_w + \delta t_{f\rho}(\mathbf{x})) = \phi(t_w) + \delta\phi(\mathbf{x}, t_w) + \dot{\phi}(t_w)\delta t_{f\rho}(\mathbf{x}) \quad (4.3)$$

$$\phi_w(\mathbf{x}, t_w + \delta t_{f\rho}(\mathbf{x})) = \phi_w(t_w) + \delta\phi_w(\mathbf{x}, t_w) + \dot{\phi}_w(t_w)\delta t_{f\rho}(\mathbf{x}), \quad (4.4)$$

where  $\delta\phi$  and  $\delta\phi_w$  are defined on the flat slice. For the unperturbed values this gives  $\phi(t_w) = \phi_w(t_w)$ . For the perturbations it gives

$$\zeta(\mathbf{x}, t_+) = H\delta t_{f\rho}(\mathbf{x}) = H \frac{\delta\phi_w(\mathbf{x}, t_w) - \delta\phi(\mathbf{x}, t_w)}{\dot{\phi}(t_w) - \dot{\phi}_w(t_w)}. \quad (4.5)$$

During hybrid inflation  $\dot{\phi} < 0$ , and we need  $\dot{\phi}(t_w) < \dot{\phi}_w(t_w)$ , or the waterfall will never start.

Now we invoke Eq. (3.3). Discounting the strong cancellation  $m^2 \simeq h^2 A^2$  it gives

$$\phi_w(\mathbf{x}, t) = \frac{1}{g}(m^2 - h^2 A^2(\mathbf{x}, t))^{1/2} \simeq \frac{m}{g} - \frac{1}{2} \frac{h^2 A^2(\mathbf{x}, t)}{mg}. \quad (4.6)$$

In most previous work,  $A$  is taken to be a scalar field. For single-field hybrid inflation [2, 3],  $\dot{\phi}_w$  is supposed to be negligible. Then  $\phi_w$  has a practically time-independent value and the waterfall slice corresponds to simply  $\phi(\mathbf{x}, t) = \phi_w(\mathbf{x})$ . For two-brid inflation [4, 5, 6],  $\phi$  and  $A$  have equal status and the time-dependence of  $\phi_w$  is significant. We have checked that in this case, Eqs. (4.5) and (4.6) are equivalent to the result (4.1) given in [5].

In our case  $A$  is the magnitude of a  $U(1)$  gauge field with the action (3.1). Let us first follow [7] by setting  $\alpha = -2$ . From Eq. (3.10), this makes  $W(\mathbf{x}, t)$  time-independent. Then, if we ignore the perturbation  $\delta f$  we have  $A(\mathbf{x}, t) \propto 1/f \propto a^2$ . This gives Eq. (3.16) for  $\zeta(\mathbf{x}, t_+)$ , with<sup>5</sup>

$$\zeta_\phi(\mathbf{x}, t_+) = \frac{\zeta_\phi(\mathbf{x})}{1 - X}, \quad (4.7)$$

$$\zeta_W(\mathbf{x}, t_+) = \frac{\hat{\zeta}_W(\mathbf{x})}{1 - X}, \quad (4.8)$$

where

$$X \equiv \frac{h^2 W^2(t_w)}{M_P m g \sqrt{2\epsilon(t_w)}}, \quad (4.9)$$

and

$$\hat{\zeta}_W(\mathbf{x}) = -(X/W^2(t_w)) \left( \mathbf{W}(t_w) \cdot \delta \mathbf{W}(\mathbf{x}, t_w) + \frac{1}{2} \delta \mathbf{W}(\mathbf{x}, t_w) \cdot \delta \mathbf{W}(\mathbf{x}, t_w) \right). \quad (4.10)$$

(We display  $t_w$  for future reference, even though we are for the moment taking  $W$  to be time-independent.) We need  $X < 1$  for the waterfall to end.

Ignoring the time-dependence of  $\mathbf{A}$  we have  $\zeta_W(\mathbf{x}, t_+) = \hat{\zeta}_W(\mathbf{x})$ , which is the result obtained in [7]. We see that  $\zeta_W$  becomes bigger when the time-dependence is taken into account. The effect of the time-dependence of  $\mathbf{A}$  was previously considered in [29], who conclude that it *decreases*  $\zeta_W$  by a factor  $e^{-2N(k)}$  making it far too small to have an observable effect. But the calculation of [29] is not from first principles because  $\mathbf{A}$  is treated as a scalar field.

We have yet to include  $\delta f$ . This has no effect on  $\zeta_W$ , but for  $\zeta_\phi$  it cancels the effect of  $\dot{A}$  giving

$$\zeta_\phi(\mathbf{x}, t_+) = \zeta_\phi(\mathbf{x}). \quad (4.11)$$

In the above we worked with  $\phi_w(A)$  defined by  $m^2(\phi_w, A) = 0$ . That allows comparison with previous work where  $A$  is a scalar field, and with [7, 29] where  $A$  is the magnitude of a gauge field. But in the latter case the calculation becomes simpler if we use instead  $\phi_w(W)$  defined by  $m^2(\phi_w, (W/f(\phi_w))) = 0$ , because it is  $\mathbf{W}$  that decouples from  $\phi$ . With our current assumption  $\mu^2 = 0$ ,  $\dot{\mathbf{W}}$  vanishes which means that  $\dot{\phi}_w(W) = 0$ . Evaluating  $\delta\phi_w(W)$  we again arrive at Eqs. (4.8) and (4.11). According to this derivation, the factor  $1 - X$  which was absent in [7] comes from  $\delta f$ .

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<sup>5</sup>We have checked that in this and the following cases, the third and fourth terms of Eq. (3.16) are negligible.

### 4.3 The varyon mechanism

Allowing  $\mu^2 \neq 0$ , we have  $\dot{W} \neq 0$  and hence  $\dot{\phi}_w(W) \neq 0$ . This gives<sup>6</sup>

$$\zeta(\mathbf{x}, t_+) = \left[ 1 + \frac{\mu^2}{6H^2} \frac{X}{1-X} \right]^{-1} \left[ \zeta_\phi(\mathbf{x}) + \frac{\hat{\zeta}_W(\mathbf{x})}{1-X} \right] \quad (4.12)$$

We see that  $\zeta_\phi(\mathbf{x}, t)$  is altered by the waterfall, which seems to contradict the statement in Section 2.2, that the slow-roll inflation result  $\zeta_\phi(\mathbf{x})$  given by Eq. (2.12) persists even after slow-roll ends. There is in fact no contradiction, because the presence of  $\mathbf{W}(t)$  means that we are not dealing with exact slow-roll inflation. As a result,  $\phi(\mathbf{x}, t)$  is not the only time-dependent field, and the effect on  $N(\mathbf{x}, t, t_*)$  of its perturbation  $\delta\phi_*$  is not removed by the time shift which makes  $\phi_*$  homogeneous. Note that the perturbation  $\delta\mathbf{W}_*(\mathbf{x})$  is in this context irrelevant. It is the quantity  $\dot{W}(t)$  that is causing the effect.

In our case  $W$  is the magnitude of a gauge field and its effect on  $\zeta$  occurs during the waterfall. But in general, any time-dependent field might do the same thing, i.e. have a negligible effect during slow-roll inflation but a significant one later owing to the time-dependence of its unperturbed part. The only exception is if the field is a slowly rolling scalar field; in that case its effect is just to slightly alter the direction in field space of the trajectory, which as we discussed in Section 2.2 will still give the (almost unchanged) the time-independent perturbation  $\zeta_\phi(\mathbf{x})$ . We propose to call a field causing the new effect a *varyon*.

The varyon mechanism can remove the bound (2.15) and hence the bound (2.16) on the tensor fraction. That might allow an observable tensor fraction within a small-field inflation model. The possibility of avoiding Eq. (2.16) was mooted in [31, 32] but they did not find a mechanism. One can easily avoid Eq. (2.16) by abandoning the canonical kinetic term for the inflaton [33] and we are now, for the first time, pointing to a possible mechanism with the canonical kinetic term.

In our case, the varyon  $\mathbf{W}$  has only a small effect except in the the very fine-tuned regime  $(1-X) \ll |\mu^2|/H^2$ . Even if its effect is significant, and reduces  $\zeta_\phi$  (positive  $\mu^2$ ) so as to give  $r > 12\epsilon$ , it cannot make  $r$  big enough to observe because the end-of-inflation formula requires  $H \lesssim 10^9$  GeV corresponding to  $r < 10^{-10}$ . It remains to be seen if a different varyon, perhaps a scalar field, can give a more interesting result.

### 4.4 Anisotropic spectrum and bispectrum

Since we are taking  $W \gg H$ , the linear term of  $\zeta_W$  dominates, leading to a spectrum of the form [7, 10]

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k) \left[ 1 - \beta \left( \hat{\mathbf{A}} \cdot \hat{\mathbf{k}} \right)^2 \right]. \quad (4.13)$$

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<sup>6</sup>In [30], this formula is given with  $1-X$  incorrectly replaced by  $1+X$ . That makes the first term must be close to 1 whatever the value of  $X$ .

On cosmological scales, observation requires  $|\beta| \lesssim 10^{-1}$ , and barring a detection PLANCK will give  $|\beta| \lesssim 10^{-2}$  [34]. We therefore have  $\mathcal{P}_\zeta(\mathbf{k}) \simeq \mathcal{P}_\zeta(k)$  with (Eq. (2.3))  $\mathcal{P}_\zeta(k) \simeq (5 \times 10^{-5})^2$ .

With the parameters constrained to give  $\beta \ll 1$ , we have [7, 10]

$$\begin{aligned} \beta &= \frac{h^4 W^2(t_w)}{m^2 g^2} \frac{\epsilon(t_k)}{\epsilon(t_w)} \\ &\times \frac{\mathcal{P}_{\zeta_\phi}(k)}{\mathcal{P}_\zeta(k)} (1 - X)^{-2} \\ &\times e^{-\frac{2\mu^2}{3H^2} N(k)} \left[ 1 + \frac{\mu^2}{6H^2} \frac{X}{1 - X} \right]^{-2}, \end{aligned} \quad (4.14)$$

where  $t_k$  is the epoch of horizon exit for the scale  $k$ , and  $\mathcal{P}_{\zeta_\phi}$  is given by Eq. (2.13). The first line is the result of [7], who assumed  $\mathcal{P}_\zeta(k) = \mathcal{P}_{\zeta_\phi}(k)$ . The first factor of the second line drops that assumption while the second factor takes account of the inhomogeneity of  $f(\phi)$ . The third line allows  $\mu \neq 0$ .

Including the quadratic term of  $\hat{\zeta}_w$  we reproduce the result of [7] for the reduced bispectrum:

$$f_{\text{NL}} = f_{\text{NL}}^{\text{iso}} (1 + f^{\text{ani}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)) \quad (4.15)$$

where

$$f^{\text{ani}} = \frac{-(\hat{\mathbf{A}} \cdot \hat{\mathbf{k}}_1)^2 - (\hat{\mathbf{A}} \cdot \hat{\mathbf{k}}_2)^2 + (\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2)(\hat{\mathbf{A}} \cdot \hat{\mathbf{k}}_1)(\hat{\mathbf{A}} \cdot \hat{\mathbf{k}}_2)}{\sum k_i^3 / k_3^3} + 2 \text{ perms.} \quad (4.16)$$

$$f_{\text{NL}}^{\text{iso}} = \frac{5}{3} \frac{\beta^2}{X}. \quad (4.17)$$

## 5. Conclusion

Although there is so far no evidence for statistical anisotropy of the primordial curvature perturbation  $\zeta$ , mechanisms have been proposed for generating it. Most of them invoke a vector field.

One mechanism takes the vector field to be homogeneous during inflation, but causes significant anisotropy in the expansion [35] (for a recent review of this approach see [36]). Then the perturbations of scalar fields generated from the vacuum fluctuation will be statistically anisotropic, and so too will be  $\zeta$  on the usual assumption that it originates from one or more of these perturbations.

A different mechanism takes the inflationary expansion to be practically isotropic, but generates a vector field perturbation from the vacuum fluctuation [7, 10] (for the most recent paper on this approach see [37]).<sup>7</sup>

In this paper we have given the first complete treatment of the version of the second mechanism proposed in [7], which couples the waterfall field to a gauge field

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<sup>7</sup>The use of a vector field to generate a contribution to  $\zeta$  was first mooted in [38].

$\mathbf{A}$  whose kinetic function  $f^2$  depends on the inflaton. We have confirmed their claim that the statistical anisotropy could easily be big enough to observe, and we have also discovered a completely new effect; if  $\mathbf{W} \equiv f\mathbf{A}$  is time-dependent it causes the usual *inflaton* contribution  $\zeta_\phi$  to vary during the waterfall. This ‘varyon’ effect might still occur if  $\mathbf{W}$  is replaced by a time-dependent (but not slowly rolling) *scalar* field and it might have nothing to do with the waterfall.

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### A. Equations of Motion for $\phi(\mathbf{x}, t)$ and $\mathbf{W}(\mathbf{x}, t)$

Extremizing the action in Eq. (3.1) with respect to fields  $\phi$ ,  $B_\mu$  and their derivatives we obtain field equations

$$[\partial_\mu + \partial_\mu \ln \sqrt{-g}] \partial^\mu \phi + V' + \frac{1}{2} f f' F_{\mu\nu} F^{\mu\nu} = 0; \quad (\text{A.1})$$

$$[\partial_\mu + \partial_\mu \ln \sqrt{-g}] f F^{\mu\nu} = 0, \quad (\text{A.2})$$

where  $g \equiv \det(g_{\mu\nu})$  and  $f' \equiv \partial f / \partial \phi$ . Choosing the temporal gauge  $B_0 = 0$  and a line element of the unperturbed universe in Eq. (1.1), one finds equations of motion for the fields  $\phi(\mathbf{x}, t)$  and  $\mathbf{B}(\mathbf{x}, t)$

$$\ddot{\phi} + 3H\dot{\phi} - a^{-2}\nabla^2\phi + V' = -\frac{1}{2} f f' F_{\mu\nu} F^{\mu\nu}, \quad (\text{A.3})$$

$$\ddot{B}_i + \left( H + 2\frac{\dot{f}}{f} \right) \dot{B}_i - a^{-2}\nabla^2 B_i = a^{-2} 2\frac{\partial_j f}{f} \partial_j B_i, \quad (\text{A.4})$$

Recasting the above equations in terms of  $\mathbf{W} \equiv f\mathbf{B}/a$  and dropping gradient terms, one arrives at equations of motion for homogeneous fields  $\phi(t)$  and  $\mathbf{W}(t)$

$$\ddot{\phi} + 3H\dot{\phi} + V' = \frac{f'}{f} \left[ \dot{\mathbf{W}} + \left( H - \frac{\dot{f}}{f} \right) \mathbf{W} \right]^2, \quad (\text{A.5})$$

$$\ddot{\mathbf{W}} + 3H\dot{\mathbf{W}} + \left( 2H^2 - H\frac{\dot{f}}{f} - \frac{\ddot{f}}{f} \right) \mathbf{W} = 0, \quad (\text{A.6})$$

where we also used  $\dot{H} \simeq 0$ .

Decomposing the field  $\mathbf{W}(\mathbf{x}, t)$  as in Eq. (3.15) and similarly the field  $\phi(\mathbf{x}, t)$ , we find equations of motion for  $\delta\phi(\mathbf{x}, t)$  and  $\delta\mathbf{W}(\mathbf{x}, t)$  from Eqs. (A.3) and (A.4). Keeping only the first order terms and switching to the Fourier space they become

$$\begin{aligned} \delta\ddot{\phi}_{\mathbf{k}} + 3H\delta\dot{\phi}_{\mathbf{k}} + \left(\frac{k^2}{a^2} + V''\right)\delta\phi_{\mathbf{k}} = \\ = 2\frac{f'}{f} \left[ \dot{\mathbf{W}} + \left(H - \frac{\dot{f}}{f}\right) \mathbf{W} \right] \left[ \delta\dot{\mathbf{W}}_{\mathbf{k}} + \left(H - \frac{\dot{f}}{f}\right) \delta\mathbf{W}_{\mathbf{k}} - \delta\left(\frac{\dot{f}}{f}\right)_{\mathbf{k}} \mathbf{W} \right] \end{aligned} \quad (\text{A.7})$$

$$\begin{aligned} \delta\ddot{\mathbf{W}}_{\mathbf{k}} + 3H\delta\dot{\mathbf{W}}_{\mathbf{k}} + \left(2H^2 - H\frac{\dot{f}}{f} - \frac{\ddot{f}}{f}\right)\delta\mathbf{W}_{\mathbf{k}} + \frac{k^2}{a^2}\delta\mathbf{W}_{\mathbf{k}} = \\ = \left[ H\delta\left(\frac{\dot{f}}{f}\right)_{\mathbf{k}} + \delta\left(\frac{\ddot{f}}{f}\right)_{\mathbf{k}} + \frac{f'}{f} \frac{k^2}{a^2} \delta\phi_{\mathbf{k}} \right] \mathbf{W}. \end{aligned} \quad (\text{A.8})$$

With our choice  $f \propto a^\alpha$  for the unperturbed  $f$ , the above expressions become

$$\begin{aligned} \delta\ddot{\phi}_{\mathbf{k}} + 3H\delta\dot{\phi}_{\mathbf{k}} + \left(\frac{k^2}{a^2} + V''\right)\delta\phi_{\mathbf{k}} = \\ = \frac{-2\alpha}{\sqrt{2\epsilon}M_{\text{P}}} \left[ \dot{\mathbf{W}} + H(1 - \alpha) \mathbf{W} \right] \left[ \delta\dot{\mathbf{W}}_{\mathbf{k}} + H(1 - \alpha) \delta\mathbf{W}_{\mathbf{k}} - \alpha \mathbf{W} \frac{H}{\dot{\phi}} \delta\dot{\phi}_{\mathbf{k}} \right] \quad (\text{A.9}) \\ \delta\ddot{\mathbf{W}}_{\mathbf{k}} + 3H\delta\dot{\mathbf{W}}_{\mathbf{k}} + \left(\frac{k^2}{a^2} + \mu^2\right)\delta\mathbf{W}_{\mathbf{k}} = \\ = \frac{-\alpha \mathbf{W}}{\sqrt{2\epsilon}M_{\text{P}}} \left[ \delta\ddot{\phi}_{\mathbf{k}} + H(1 + 2\alpha) \delta\dot{\phi}_{\mathbf{k}} + \frac{k^2}{a^2} \delta\phi_{\mathbf{k}} \right], \end{aligned} \quad (\text{A.10})$$

The energy density of the vector field in Eq. (3.1) is given by [39]  $\rho_B(\mathbf{x}, t) = -f^2 F_{\mu\nu} F^{\mu\nu}/4$ . From this it is easy to see that the background value of  $\rho_B(\mathbf{x}, t)$  is given by

$$\rho_B(t) = \frac{1}{2} f^2 \left( \frac{\dot{B}}{a} \right)^2 = \frac{1}{2} \left[ \dot{W} + \left(H - \frac{\dot{f}}{f}\right) W \right]^2 \simeq \frac{1}{2} H^2 W^2. \quad (\text{A.11})$$

The right hand side of Eq.(A.5) is negligible if  $\rho_B$  satisfies Eq. (3.12). We now show that the same is true of the right hand sides of Eqs. (A.9) and (A.10). At the epoch  $k \sim aH$ , the terms on the left hand sides are of order  $H^3$  and Eq. (3.12) ensures that the right hand sides are indeed much smaller. At the epoch  $aH/k = \exp(N_k(t)) \gg 1$ , the first term of each left hand side is negligible. The other two terms are of order  $|\eta| \equiv |V''|/3H^2$  for Eq. (A.9) and of order  $|\eta_W| \equiv |\mu^2|/3H^2$  for Eq. (A.10). Eq. (3.12) ensures that the right hand side of Eq. (A.9) is negligible, and it ensures that the right hand side of Eq. (A.10) is negligible if also  $|\eta_W| \gg 10^{-5}$ . But the latter condition is irrelevant, because its violation makes the time-dependence of  $W$  (coming then from the right hand side) negligible.

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